Name: ______________________________ Date: _______________ Period: __________

Pythagorean Theorem

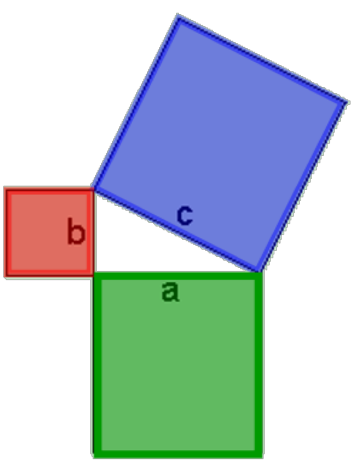
The longest side of the triangle is called the "hypotenuse"

THEOREM:

In a **<u>right triangle</u>**, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.
($c^2 = a^2 + b^2$)

It is believed that this theorem was known, in some form, long before the time of Pythagoras. A thousand years before Pythagoras, the Babylonians recorded their knowledge of the theorem on clay tablets. The Egyptians used the concept of the theorem in the building of the pyramids. In 1100 B.C. China, Tschou-Gun knew of this theorem.

It was, however, Pythagoras who generalized the theorem to all right triangles and is credited with its first geometrical demonstration. Consequently, the theorem bears his name.



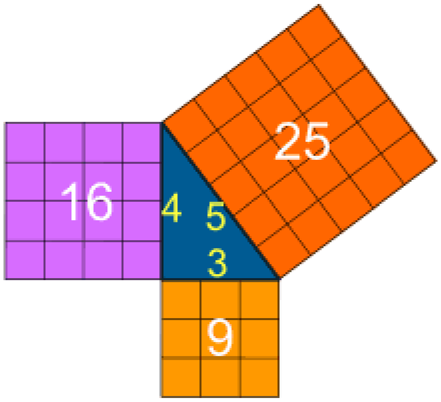
Geometrical Proof: The Pythagorean Theorem has drawn a good deal of attention from mathematicians. There are hundreds of geometrical proofs (or demonstrations) of the theorem, with even a larger number of algebraic proofs.

Geometrically, the Pythagorean Theorem can be interpreted as discussing the areas of squares whose sides are the sides of the triangle (as seen in the picture at the left). The theorem can be rephrased as, "*The (area of the) square described upon the hypotenuse of a right triangle is equal to the sum of the (areas of)the squares described upon the other two sides.*"

Sure ... ?

Let's see if it really works using an example.

Example: A "3,4,5" triangle has a right angle in it.



Let's check if the areas **are** the same:

$3^2 + 4^2 = 5^2$

Calculating this becomes:

$9 + 16 = 25$

It works ... like Magic!

Converse:

Theorem: If a triangle is a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

Converse: If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, the triangle is a right triangle.

Corollary: A corollary of the Pythagorean theorem's converse is a simple means of determining whether a triangle is **right**, **obtuse**, or **acute**, as follows.

Let *c* be chosen to be the longest of the three sides and $a + b > c$ (otherwise there is no triangle according to the triangle inequality). The following statements apply:

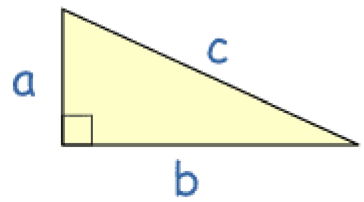
- **If $c^2 = a^2 + b^2$, then the triangle is right.**
- **If $c^2 < a^2 + b^2$, then the triangle is acute.**
- **If $c^2 > a^2 + b^2$, then the triangle is obtuse.**

Why Is This Useful?

If we know the lengths of **two sides** of a right angled triangle, we can find the length of the **third side**. (But remember it only works on right angled triangles!)

How Do I Use it?

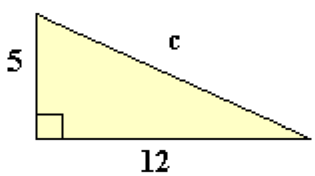
Write it down as an equation:



$a^2 + b^2 = c^2$

Now you can use algebra to find any missing value, as in the following examples:

Example: Solve this triangle.



$a^2 + b^2 = c^2$

$5^2 + 12^2 = c^2$

$25 + 144 = c^2$

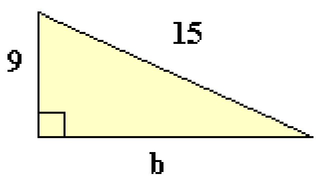
$169 = c^2$

$c^2 = 169$

$c = \sqrt{169}$

$c = 13$

Example: Solve this triangle.



$a^2 + b^2 = c^2$

$9^2 + b^2 = 15^2$

$81 + b^2 = 225$

Take 81 from both sides:

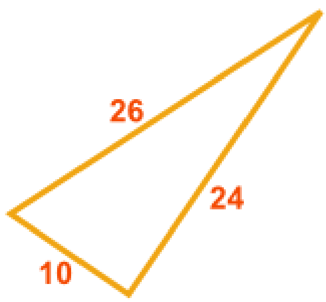
$b^2 = 144$

$b = \sqrt{144}$

$b = 12$

It works the other way around, too: when the three sides of a triangle make $c^2 = a^2 + b^2$, then the triangle is right angled.

Example: Does this triangle have a Right Angle?



Does $a^2 + b^2 = c^2$?

- $a^2 + b^2 = 10^2 + 24^2 = 100 + 576 =$ **676**
- $c^2 = 26^2 =$ **676**

They are equal, so ...

Yes, it does have a Right Angle!

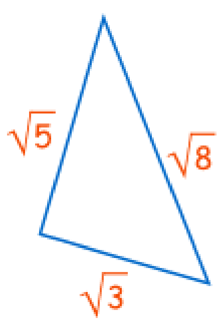
Example: Does an 8, 15, 16 triangle have a Right Angle?

Does 8^2 + **15**^{2} = **16**^{2} ?

- $8^2 + 15^2 = 64 + 225 =$ **289**,
- but $16^2 =$ **256**

So, NO, it does not have a Right Angle

Example: Does this triangle have a Right Angle?



Does $a^2 + b^2 = c^2$?

Does $(\sqrt{3})^2 + (\sqrt{5})^2 = (\sqrt{8})^2$?
Does $3 + 5 = 8$?

Yes, it does!

So this **is** a right-angled triangle